A Trajectory Tracking Robust Controller of Surface Vessels With Disturbance Uncertainties

Yang Yang, Jialu Du, Hongbo Liu, Chen Guo, and Ajith Abraham

***Abstract*— This brief considers the problem of trajectory tracking control for marine surface vessels with unknown time-variant environmental disturbances. The adopted mathematical model of the surface ship movement includes the Coriolis and centripetal matrix and the nonlinear damping terms. An observer is constructed to provide an estimation of unknown disturbances and is applied to design a novel trajectory tracking robust controller through a vectorial backstepping technique. It is proved that the designed tracking controller can force the ship to track the arbitrary reference trajectory and guarantee that all the signals of the closed-loop trajectory tracking control system of ships are globally uniformly ultimately bounded. The simulation results and comparisons illustrate the effectiveness of the proposed controller and its robustness to external disturbances.**

***Index Terms*— Disturbance observer, nonlinear, robust, trajectory tracking control of vessels, vectorial backstepping.**

I. Introduction

TRAJECTORY tracking control of surface vessels is an important control problem. It is of great significance for navigation in safety, energy saving, and emission reduction. It has attracted a great deal of attention from the control community both in theory and in practice [1]. In [2], a simplified linear model was used to develop an adaptive high precision track controller for ships through a combination of feed forward and linear-quadratic-Gaussian feedback control. In fact, the tracking control for a ship has an inherently nonlinear character. Taking advantage of the model free intelligent control techniques, [3] presented a fuzzy proportional–integral-derivative track autopilot for ships, and [4] developed a neural network trajectory tracking controller for ships. In recent years, several significant results have been presented through applying nonlinear control techniques to the nonlinear maneuvering mathematical models of ships. Jiang [5] proposed two global tracking control laws for underactuated vessels using Lyapunov's direct method. Petterson and Nijmeijer [6] illustrated a semiglobal exponential stabilization of the tracking error for any desired trajectory using an integrator backstepping approach. Furthermore, they developed an exponential trajectory tracking control law for the ship based on a coordinate transformation and integrator backstepping with the aid of tracking control of chained form systems. The effectiveness of the control law was validated by the experimental results on a scale 1:70 model of an offshore supply vessel in the laboratory [7]. Yu *et al.* [8] introduced the second-level sliding mode surface approach to design a trajectory tracking control law for an underactuated ship with parameter uncertainties. Wondergem *et al.* [9] presented an observer-controller output feedback trajectory tracking control scheme with a semiglobal exponential stability for fully actuated surface ships in the presence of the Coriolis and centripetal matrix and the nonlinear damping terms.

On the other hand, the ships in the sea are always exposed to the environmental disturbances induced by wind, waves, and ocean currents. It is necessary to develop robust controllers for external disturbances. Under constant disturbances, a nonlinear trajectory tracking control law was designed for a fully actuated ship simultaneously considering the Coriolis and centripetal matrix and the nonlinear damping terms in [10]. Aschemann and Rauh [11] presented two alternative nonlinear control approaches to track the trajectories through the extended linearization technique, where the tracking accuracy was improved significantly by introducing a compensating control action provided by a disturbance observer for constant disturbances. Using the backstepping technique, a discontinuous feedback control law [12] and a new family of smooth time-varying dynamic feedback laws [13] have been derived for underactuated surface vessels, respectively.

In general, the mathematical model of ships does not simultaneously consider the Coriolis and centripetal matrix and the nonlinear damping terms, or uncertain time-variant environmental disturbances are not dealt with during the control design procedures. The sea state is, however, constantly changing during the navigation of ships. For underactuated ships, Do [14] provided a solution for the practical stabilization through several nonlinear coordinate changes, the transverse function approach, the backstepping technique, the Lyapunov's direct method, and usage of the ship dynamics.

For fully actuated surface vessels, this brief presents a novel approach to solve the trajectory tracking control problem. The mathematical model of the ship movement simultaneously contains the Coriolis and centripetal matrix and the nonlinear damping terms. The disturbances induced by wind, waves, and currents are considered. Our proposed approach is featured with a disturbance observer that

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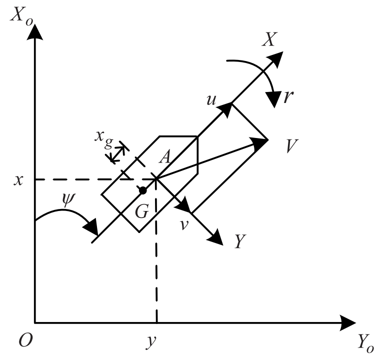

Fig. 1. Definition of the earth-fixed OX_oY_o and the body-fixed AXY coordinate frames.

is introduced to estimate the time-variant uncertain environmental disturbances.

II. Problem Formulation

Definition of the reference coordinate frames of ship motion is shown in Fig. 1, where OX_oY_o is the earth-fixed frame and AXY is the body-fixed frame. The coordinate origin O of the earth-fixed reference frame OX_oY_o is the original position of the desired trajectory. The axis OX_o is directed to the North and OY_o is directed to the East. The coordinate origin A of the body-fixed frame is taken as the geometric center point of the ship structure. The axis AX is directed from aft to fore, the axis AY is directed to starboard, and the normal axis AZ is directed from top to bottom. Under the assumption that the ship is port–starboard symmetric, the gravity center G is located a distance x_g between the gravity center of the ship and the origin of the body-fixed frame along axis AX. The vector $\eta = [x, y, \psi]^T$ is the actual track of the ship in the earth-fixed frame, consisting of the ship position (x, y) and yaw angle $\psi \in [0, 2\pi]$. The vector $\nu = [u, \upsilon, r]^T$ is the velocity vector of the ship in the body-fixed frame. The variables u, υ, and r are, respectively, the forward velocity (surge), the transverse velocity (sway), and the angular velocity in yaw of the ship. Surge is decoupled from sway and yaw. Neglecting the motions in heave, pitch and roll, the 3-DOF nonlinear motion equations of a surface ship can be expressed as [15]

$$\dot{\eta} = R(\psi)\nu \tag{1}$$

$$M\dot{\nu} + C(\nu)\nu + D(\nu)\nu = \tau + b \tag{2}$$

where $\tau = [\tau_1, \tau_2, \tau_3]^T$ is the control input vector, $b(t) = [b_1(t), b_2(t), b_3(t)]^T$ is the vector representing unknown and time-variant external environmental disturbances due to wind, waves, and ocean currents in the body-fixed frame. Here, it is assumed that the changing rate of disturbances is bounded, i.e., $\| \dot{b}(t) \| \leqslant C_d < \infty$, where C_d is a nonnegative constant. The above assumption is reasonable because environmental energy applied to the ship is limited. The matrix $R(\psi)$ is rotation matrix defined as

$$R(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

with the property $R^{-1}(\psi) = R^T(\psi)$. Here, M is nonsingular, symmetric, and positive definite inertia matrix, $C(\nu)$ is the matrix of Coriolis and centripetal terms, and $D(\nu)$ is the damping matrix. They are, respectively

$$M = \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & m_{23} \\ 0 & m_{32} & m_{33} \end{bmatrix} \tag{4}$$

$$C(\nu) = \begin{bmatrix} 0 & 0 & -m_{22}\upsilon - m_{23}r \\ 0 & 0 & m_{11}u \\ m_{22}\upsilon + m_{23}r & -m_{11}u & 0 \end{bmatrix} \tag{5}$$

$$D(\nu) = \begin{bmatrix} d_{11}(u) & 0 & 0 \\ 0 & d_{22}(\upsilon, r) & d_{23}(\upsilon, r) \\ 0 & d_{32}(\upsilon, r) & d_{33}(\upsilon, r) \end{bmatrix}. \tag{6}$$

In (4)–(6)

$$\begin{aligned}
m_{11} &= m - X_{\dot{u}} \\
m_{22} &= m - Y_{\dot{\upsilon}} \\
m_{23} &= mx_g - Y_{\dot{r}} \\
m_{32} &= mx_g - N_{\dot{\upsilon}} \\
m_{33} &= I_z - N_{\dot{r}} \\
d_{11}(u) &= -X_u - X_{|u|u}|u| \\
d_{22}(\upsilon, r) &= -Y_\upsilon - Y_{|\upsilon|\upsilon}|\upsilon| - Y_{|r|\upsilon}|r| \\
d_{23}(\upsilon, r) &= -Y_r - Y_{|\upsilon|r}|\upsilon| - Y_{|r|r}|r| \\
d_{32}(\upsilon, r) &= -N_\upsilon - N_{|\upsilon|\upsilon}|\upsilon| - N_{|r|\upsilon}|r| \\
d_{33}(\upsilon, r) &= -N_r - N_{|\upsilon|r}|\upsilon| - N_{|r|r}|r|
\end{aligned}$$

where m is the mass of the ship, I_z is the moment of inertia about the yaw rotation, and the other symbols, for example, $Y_{\dot{u}} = \partial Y / \partial \dot{u}$, are referred to as hydrodynamic derivatives. The reader may refer to [16] for more details.

The control objective in this brief is to design a feedback control law τ for (1) and (2) such that the position and yaw angle $\eta(t)$ of ships tracks arbitrary smooth reference trajectory $\eta_d(t)$, while it is guaranteed that all the signals of the resulting closed-loop trajectory tracking system of a ship are globally uniformly ultimately bounded.

Assumption 1: The desired smooth reference signal η_d is bounded and has the bounded first and second time derivatives $\dot{\eta}_d$ and $\ddot{\eta}_d$.

III. Controller Design

In this section, a disturbance observer is designed to estimate the unknown time-variant external environmental disturbances of (1) and (2). Then, we present the robust trajectory tracking controller for ships that solves the control objective as stated in Section II. The closed-loop trajectory tracking control system of a ship mainly consists of two parts: 1) the ship subjected to external disturbances and 2) the trajectory tracking controller with the disturbance observer. The schematic diagram is shown in Fig. 2.

A. Disturbance Observer Design

Using the exponential convergent observer for a general nonlinear system from [14], we construct the disturbance observer for the disturbance vector b of (1) and (2) as follows:

$$\hat{b} = \beta + K_0 M\nu \tag{7}$$

$$\dot{\beta} = -K_0\beta - K_0[-C(\nu)\nu - D(\nu)\nu + \tau + K_0 M\nu] \tag{8}$$

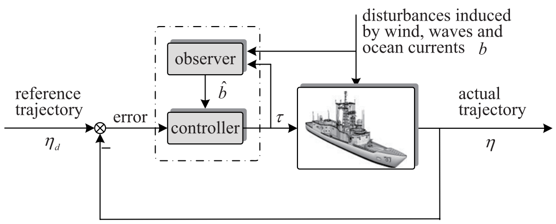

Fig. 2. Diagram of the trajectory tracking control system of a ship.

where $\hat{b} = [\hat{b}_1, \hat{b}_2, \hat{b}_3]^T$ is a disturbance estimation, K_0 is a 3-by-3 positive definite symmetric observer gain matrix, and β is a 3-D intermediate auxiliary vector.

Define the estimation error vector $\tilde{b} = [\tilde{b}_1,\ \tilde{b}_2,\ \tilde{b}_3]^T$ of disturbance vector b as

$$\tilde{b} = b - \hat{b}. \tag{9}$$

From (2), (7), and (8), we have

$$\begin{aligned}\dot{\hat{b}} &= \dot{\beta} + K_0 M \dot{\nu}\\ &= -K_0\beta - K_0[-C(\nu)\nu - D(\nu)\nu + \tau + K_0 M\nu]\\ &\quad + K_0[-C(\nu)\nu - D(\nu)\nu + \tau + b]\\ &= K_0[b - (\beta + K_0 M\nu)]\\ &= K_0(b - \hat{b}).\end{aligned} \tag{10}$$

Then, the derivative of (9) is

$$\dot{\tilde{b}} = \dot{b} - K_0(b - \hat{b}) = \dot{b} - K_0\tilde{b}. \tag{11}$$

Consider the following Lyapunov function candidate:

$$V_e = \frac{1}{2}\tilde{b}^T\tilde{b}. \tag{12}$$

The time derivative of V_e along the solution of (11) is

$$\dot{V}_e = \tilde{b}^T(-K_0\tilde{b} + \dot{b}) = -\tilde{b}^T K_0\tilde{b} + \tilde{b}^T\dot{b}. \tag{13}$$

According to the following complete square inequality:

$$\tilde{b}^T\dot{b} \le \varepsilon\tilde{b}^T\tilde{b} + \frac{1}{4\varepsilon}\dot{b}^T\dot{b} \tag{14}$$

where ε is a small positive constant, (13) can be rewritten as

$$\begin{aligned}\dot{V}_e &\le -\lambda_{\min}(K_0)\tilde{b}^T\tilde{b} + \varepsilon\tilde{b}^T\tilde{b} + \frac{1}{4\varepsilon}\dot{b}^T\dot{b}\\ &\le -2[\lambda_{\min}(K_0) - \varepsilon]V_e + \frac{C_d^2}{4\varepsilon}\\ &\le -\alpha V_e + c\end{aligned} \tag{15}$$

where

$$c = \frac{C_d^2}{4\varepsilon} \tag{16}$$

$$\alpha = 2[\lambda_{\min}(K_0) - \varepsilon] \tag{17}$$

$$\lambda_{\min}(K_0) - \varepsilon > 0 \tag{18}$$

and $\lambda_{\min}(\cdot)$ represents the smallest eigenvalue of a matrix. Therefore, we have the following theorem.

Theorem 1: The disturbance observer (7) and (8) guarantees that the disturbance estimation error $\tilde{b}$ exponentially converges to a ball Ω_b centered at the origin with the radius $R_d = C_d/[2\sqrt{\varepsilon(\lambda_{\min}(K_0) - \varepsilon)}]$. The estimation error $\tilde{b}$ of disturbances can be made arbitrarily small by appropriately adjusting the design matrix K_0 and parameter ε satisfying the condition (18).

Proof: Solving (15), we have

$$0 \le V_e(t) \le \frac{c}{\alpha} + \left[V_e(0) - \frac{c}{\alpha}\right]e^{-\alpha t}. \tag{19}$$

It is known from (19) that V_e is ultimately bounded and exponentially converges to a ball centered at the origin with the radius $R_V = C_d^2/[8\varepsilon(\lambda_{\min}(K_0) - \varepsilon)]$. Furthermore, it is known from the definition of V_e that the disturbance estimation error $\tilde{b}$ exponentially converges to a ball Ω_b centered at the origin with the radius $R_d = C_d/[2\sqrt{\varepsilon(\lambda_{\min}(K_0) - \varepsilon)}]$. Therefore, the theorem is proved. ∎

Remark 1: In the case $C_d = 0$, i.e., the disturbance vector is unknown constant vector, the disturbance observer is exponentially stable. The disturbance estimation error $\tilde{b}$ exponentially converges to zero.

B. Control Law Design

Let the desired position and yaw angle of ships be $\eta_d = [x_d,\ y_d,\ \psi_d]^T$. First define the error vectors as follows:

$$\eta_e = \eta - \eta_d \tag{20}$$

$$\mathcal{X}_e = \nu - \mathcal{X}_1 \tag{21}$$

where $\mathcal{X}_1$ is the stabilization function vector of subsystem (2), ν is taken as the virtual control input vector. The control law design consists of two steps.

Step 1: Consider the following Lyapunov function candidate:

$$V_1 = \frac{1}{2}\eta_e^T\eta_e. \tag{22}$$

The derivative of η_e is given by

$$\dot{\eta}_e = \dot{\eta} - \dot{\eta}_d = R(\psi)\mathcal{X}_e + R(\psi)\mathcal{X}_1 - \dot{\eta}_d. \tag{23}$$

Then the time derivative of V_1 along the solution of (23) is

$$\dot{V}_1 = \eta_e^T\dot{\eta}_e = \eta_e^T[R(\psi)\mathcal{X}_1 - \dot{\eta}_d] + \eta_e^T R(\psi)\mathcal{X}_e. \tag{24}$$

We choose the stabilization function vector

$$\mathcal{X}_1 = R^{-1}(\psi)(-C_1\eta_e + \dot{\eta}_d) \tag{25}$$

where C_1 is a 3-by-3 positive definite symmetric design parameter matrix.

Substituting (25) into (24) yields

$$\begin{aligned}\dot{V}_1 &= \eta_e^T[R(\psi)R^{-1}(\psi)(-C_1\eta_e + \dot{\eta}_d) - \dot{\eta}_d] + \eta_e^{\mathrm{TR}}(\psi)\mathcal{X}_e\\ &= -\eta_e^T C_1\eta_e + \eta_e^{\mathrm{TR}}(\psi)\mathcal{X}_e.\end{aligned} \tag{26}$$

The coupling term $\eta_e^{\mathrm{TR}}(\psi)\mathcal{X}_e$ will be cancelled in the next step.

Step 2: From (2) and (21), we have

$$\begin{aligned}\dot{\mathcal{X}}_e &= \dot{\nu} - \dot{\mathcal{X}}_1\\ &= M^{-1}[-C(\nu)\nu - D(\nu)\nu + \tau + b - M\dot{\mathcal{X}}_1].\end{aligned} \tag{27}$$

Consider the augmented Lyapunov function candidate

$$V_2 = V_1 + \frac{1}{2}\mathcal{X}_e^{\mathrm{TM}}\mathcal{X}_e + \frac{1}{2}\tilde{b}^T\tilde{b}. \tag{28}$$

In terms of (11), (26), and (27), the time derivative of V_2 is

$$\dot{V}_2 = \dot{V}_1 + \mathcal{X}_e^{\mathrm{TM}}\dot{\mathcal{X}}_e + \tilde{b}^T\dot{\tilde{b}}$$
$$= -\eta_e^T C_1\eta_e + \mathcal{X}_e^T[R^T(\psi)\eta_e - C(\nu)\nu - D(\nu)\nu + \tau + b - M\dot{\mathcal{X}}_1] - \tilde{b}_0^{\mathrm{TK}}\tilde{b} + \tilde{b}^T\dot{b}. \quad (29)$$

We design the control input vector as

$$\tau = C(\nu)\nu + D(\nu)\nu + M\dot{\mathcal{X}}_1 - R^T(\psi)\eta_e - C_2\mathcal{X}_e - \hat{b} \quad (30)$$

where C_2 is a 3-by-3 positive definite symmetric design parameter matrix.

According to (20) and the property $R^{-1}(\psi) = R^T(\psi)$, we calculate the derivative of $\mathcal{X}_1$ as follows:

$$\dot{\mathcal{X}}_1 = \dot{R}^T(\psi)[-C_1(\eta - \eta_d) + \dot{\eta}_d] + R^T(\psi)[-C_1(\dot{\eta} - \dot{\eta}_d) + \ddot{\eta}_d]. \quad (31)$$

In addition, we have from (3)

$$\dot{R}(\psi) = \begin{bmatrix} -r\sin\psi & -r\cos\psi & 0 \\ r\cos\psi & -r\sin\psi & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & -r & 0 \\ r & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = R(\psi)S(r) \quad (32)$$

where

$$S(r) = \begin{bmatrix} 0 & -r & 0 \\ r & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Then, we obtain

$$\dot{\mathcal{X}}_1 = [R(\psi)S(r)]^T[-C_1(\eta - \eta_d) + \dot{\eta}_d] + R^T(\psi)[-C_1(\dot{\eta} - \dot{\eta}_d) + \ddot{\eta}_d]. \quad (33)$$

By substituting (7), (20), (21), and (33) into (30), (30) can be rewritten as

$$\tau = -(MS^TR^TC_1 + R^T + C_2R^TC_1)(\eta - \eta_d) + MR^T\ddot{\eta}_d + (MS^TR^T + MR^TC_1 + C_2R^T)\dot{\eta}_d + [C(\nu) + D(\nu) - MR^TC_1R - C_2 - K_0M]\nu - \beta. \quad (34)$$

Substituting (30) into (29) results in

$$\dot{V}_2 = -\eta_e^TC_1\eta_e + \mathcal{X}_e^T[R^T(\psi)\eta_e - C(\nu)\nu - D(\nu)\nu + C(\nu)\nu + D(\nu)\nu + M\dot{\mathcal{X}}_1 - R(\psi)\eta_e - C_2\mathcal{X}_e - \hat{b} + b - M\dot{\mathcal{X}}_1] - \tilde{b}^TK_0\tilde{b} + \tilde{b}^T\dot{b}$$
$$= -\eta_e^TC_1\eta_e - \mathcal{X}_e^TC_2\mathcal{X}_e + \mathcal{X}_e^T\tilde{b} - \tilde{b}^TK_0\tilde{b} + \tilde{b}^T\dot{b}. \quad (35)$$

Considering (14) and the following complete square inequalities:

$$\mathcal{X}_e^T\tilde{b} \le \varepsilon_1\mathcal{X}_e^T\mathcal{X}_e + \frac{1}{4\varepsilon_1}\tilde{b}^T\tilde{b} \quad (36)$$
$$-\mathcal{X}_e^TC_2\mathcal{X}_e \le -\lambda_{\min}(C_2M^{-1})\mathcal{X}_e^{\mathrm{TM}}\mathcal{X}_e \quad (37)$$

where ε_1 is a small positive constant, (35) can be rewritten as

$$\dot{V}_2 \le -\lambda_{\min}(C_1)\eta_e^T\eta_e - \lambda_{\min}(C_2M^{-1})\mathcal{X}_e^{\mathrm{TM}}\mathcal{X}_e + \varepsilon_1\mathcal{X}_e^T\mathcal{X}_e + \frac{1}{4\varepsilon_1}\tilde{b}^T\tilde{b} - \lambda_{\min}(K_0)\tilde{b}^T\tilde{b} + \varepsilon\tilde{b}^T\tilde{b} + \frac{1}{4\varepsilon}\dot{b}^T\dot{b}$$
$$\le -2\min\left[\lambda_{\min}(C_1), \lambda_{\min}(C_2M^{-1}) - \varepsilon_1\lambda_{\max}(M^{-1})\right. \left. \lambda_{\min}(K_0) - \frac{1}{4\varepsilon_1} - \varepsilon\right]V_2 + \frac{1}{4\varepsilon}C_d^2 \quad (38)$$

where

$$\lambda_{\min}(C_2M^{-1}) - \varepsilon_1\lambda_{\max}(M^{-1}) > 0 \quad (39)$$
$$\lambda_{\min}(K_0) - \frac{1}{4\varepsilon_1} - \varepsilon > 0 \quad (40)$$

and $\lambda_{\max}(\cdot)$ represents the largest eigenvalue of a matrix. Therefore, there is the following theorem.

Theorem 2: Under Assumption 1, for the 3-DOF nonlinear motion mathematical model of ships with unknown time-variant disturbances given by (2) and (2), the control input vector τ described by (34) together with (8) guarantees that the actual trajectory of ships tracks the arbitrary reference trajectory with the desired accuracy and all the signals of the closed-loop trajectory tracking system of ships are globally uniformly ultimately bounded by appropriately choosing the design parameter matrices C_1, C_2, and K_0 satisfying the conditions (39) and (40).

Proof: Notate

$$\mu = \min\left[\lambda_{\min}(C_1), \lambda_{\min}(C_2M^{-1}) - \varepsilon_1\lambda_{\max}(M^{-1}), \lambda_{\min}(K_0) - \frac{1}{4\varepsilon_1} - \varepsilon\right] \quad (41)$$
$$\sigma = \frac{C_d^2}{4\varepsilon}. \quad (42)$$

Then (38) can be rewritten as

$$\dot{V}_2(t) \le -2\mu V_2(t) + \sigma. \quad (43)$$

Solving the above inequality, we have

$$0 \le V_2(t) \le \frac{\sigma}{2\mu} + \left[V_2(0) - \frac{\sigma}{2\mu}\right]e^{-2\mu t}. \quad (44)$$

It is observed from (44) that $V_2(t)$ is globally uniformly ultimately bounded. Hence, η_e, $\mathcal{X}_e$, and $\tilde{b}$ are globally uniformly ultimately bounded according to (28), then $\mathcal{X}_1$ and ν are globally uniformly ultimately bounded. From the boundedness of η_d and b, we know that η and $\hat{b}$ are bounded.

From (28) and (44), we can obtain

$$\|z_1\| \le \sqrt{\frac{\sigma}{\mu} + 2\left[V_2(0) - \frac{\sigma}{2\mu}\right]e^{-2\mu t}}. \quad (45)$$

It follows that, for any $\mu_{z_1} > \sqrt{\sigma/\mu}$, there exists a constant $T_{z_1} > 0$, such that $\|z_1\| \le \mu_{z_1}$ for all $t > T_{z_1}$. Therefore, the trajectory tracking error z_1 of the ship can converge to the compact set $\Omega_{ze} := \{z_1 \in \mathbb{R}^3 | \, \|z_1\| \le \mu_{z_1}\}$. Since $\sqrt{\sigma/\mu}$ can be made arbitrarily small if the design parameters C_1, C_2, and K_0 are appropriately chosen, the actual trajectory of the ship can track the arbitrary reference trajectory with the desired accuracy. Theorem 2 is thus proved. ■

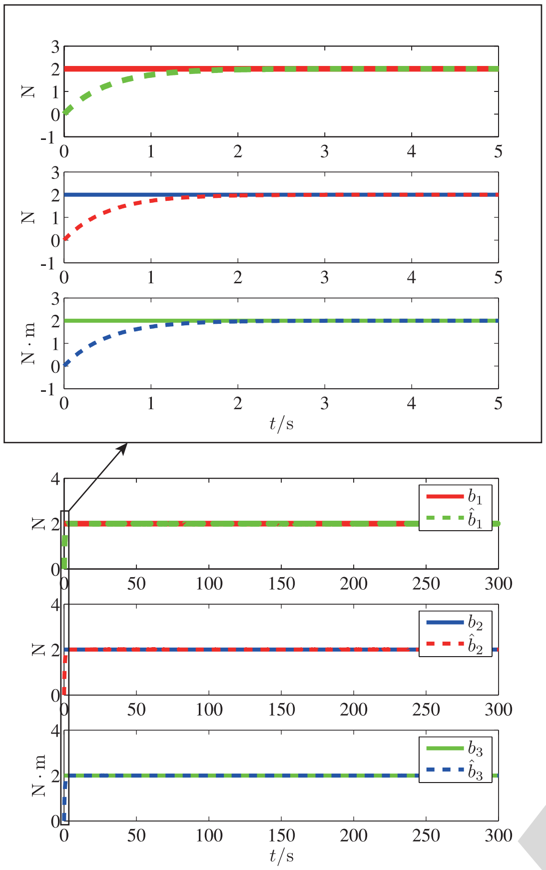

Fig. 3. Constant external disturbances b_1, b_2, b_3 and their estimations $\hat{b}_1$, $\hat{b}_2$, $\hat{b}_3$.

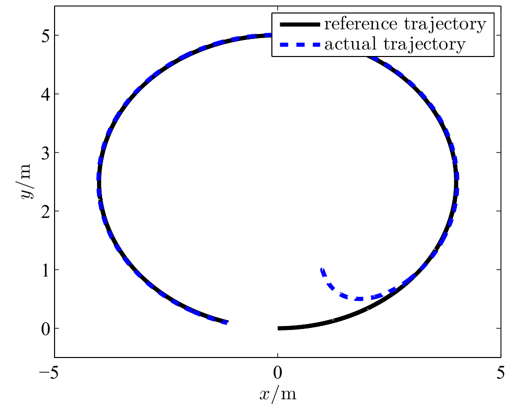

Fig. 4. Actual and reference trajectories in xy-plane under constant disturbances.

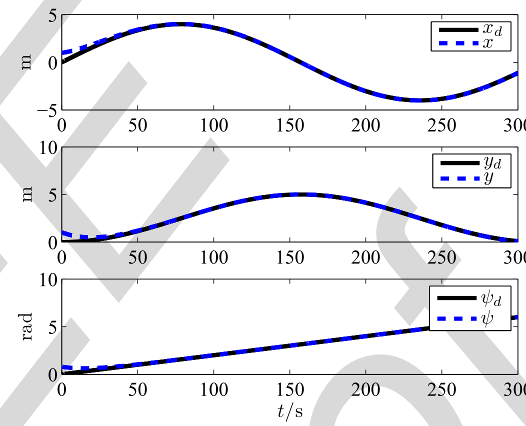

Fig. 5. Desired and actual positions and yaw angles under constant disturbances.

IV. Simulations and Comparisons

In this section, the simulation studies are carried out on CyberShip II, which is a 1:70 scale replica of a supply ship of the Marine Cybernetics Laboratory in Norwegian University of Science and Technology. The ship has the length of 1.255 m, mass of 23.8 kg, and other parameters of the ship are given in detail in [17].

We carry out the simulations with two different disturbances. In the simulations, the reference trajectory is chosen as follows:

$$\begin{aligned} x_d &= 4\sin(0.02t) \\ y_d &= 2.5(1-\cos(0.02t)) \\ \psi_d &= 0.02t \end{aligned} \tag{46}$$

which is an ellipse.

A. Trajectory Tracking Under Constant Disturbances

In this section, the disturbance vector is set as $b = [2\ \text{N}, 2\ \text{N}, 2\ \text{N}\cdot\text{m}]^T$, which corresponds to the environmental disturbances due to slowly varying wind, waves, and currents. Assume the initial conditions of the system are $[x(0), y(0), \psi(0), u(0), \upsilon(0), r(0)]^T = [1\ \text{m}, 1\ \text{m}, \pi/4\ \text{rad}, 0\ \text{m/s}, 0\ \text{m/s}, 0\ \text{rad/s}]^T$ and the initial state of the disturbance observer is $\hat{b}(0) = [0, 0, 0]^T$. The design parameter matrices are taken as $C_1 = \text{diag}[0.05, 0.05, 0.05]$, $C_2 = \text{diag}[120, 120, 120]$, $K_0 = \text{diag}[2, 2, 2]$ such that the conditions (40) and (40) are satisfied for $0.125 < \varepsilon_1 < 9.6509$ and $0 < \varepsilon < 1.9741$. The results are shown in Figs. 3–7. The external disturbances b and its estimate value $\hat{b}$ are shown in Fig. 3 from which it is clearly observed that the disturbance observer provides the rapidly exponentially convergent estimation of unknown disturbances within about 1.5 s as proved in Theorem 1. From Fig. 4, it is observed that the proposed controller is able to force the ship to track the reference trajectory. Furthermore, the curves of the desired and actual positions and yaw angles are shown in Fig. 5, which shows that the actual ship position (x, y) and yaw angle ψ can track the desired trajectory $\eta_d = [x_d, y_d, \psi_d]^T$ at a good precision in around 40 s. The curves of the surge velocity u, sway velocity υ and yaw rate r versus time are shown in Fig. 6. The corresponding control inputs are presented in Fig. 7, which shows that the control force and torque are smooth and reasonable. These results reveal that all the signals of the closed-loop trajectory tracking system of ships are globally uniformly ultimately bounded as proved in Theorem 2. Therefore, the proposed trajectory tracking controller is effective for the ship with uncertain constant disturbances.

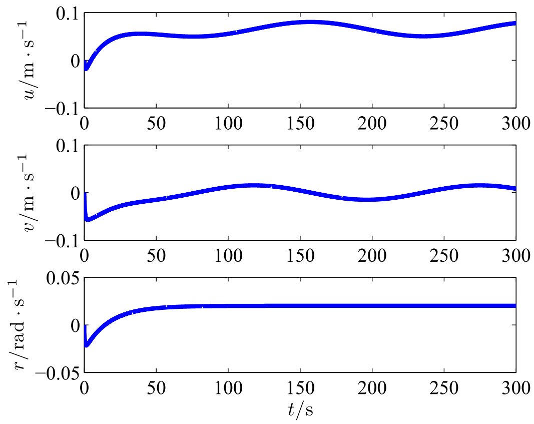

Fig. 6. Surge velocity u, sway velocity v, and yaw rate r under constant disturbances.

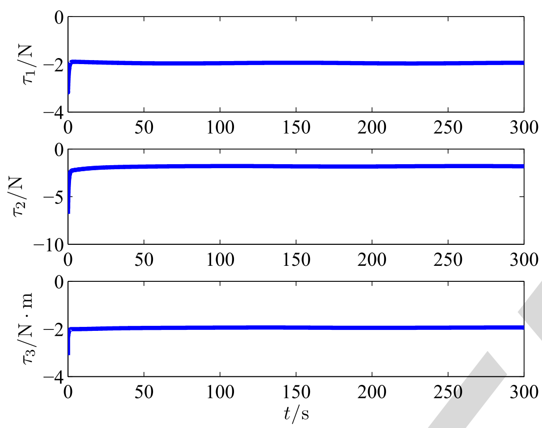

Fig. 7. Surge control force τ_1, sway control force τ_2, and yaw control torque τ_3 under constant disturbances.

B. Trajectory Tracking Under Time-Variant Disturbances

In this section, the disturbance vector is set as

$$
\begin{aligned}
b(t) &= [b_1(t), b_2(t), b_3(t)]^T \\
&= \begin{bmatrix} 1.3+\ 2.0\sin(0.02t) + 1.5\sin(0.1t)\ \text{N} \\ -0.9 + 2.0\sin(0.02t - \pi/6) + 1.5\sin(0.3t)\ \text{N} \\ -\sin(0.09t + \pi/3) - 4\sin(0.01t)\ \text{N}\cdot\text{m} \end{bmatrix}.
\end{aligned}
$$

The initial conditions of the system and the design parameters of controller are same as the counterparts in the first case of Section III-A. The results are shown in Figs. 8–12, which exhibit almost the same control performance as under constant disturbances despite the time-variant disturbances. It is obvious that the designed controller is effective when the ship is exposed to both unknown constant and time-variant disturbances, which demonstrates that the proposed controller is robust against unknown environmental disturbances.

C. Performance Comparisons

In this section, we compare the tracking performance of the designed controller (34) in this brief with the controller

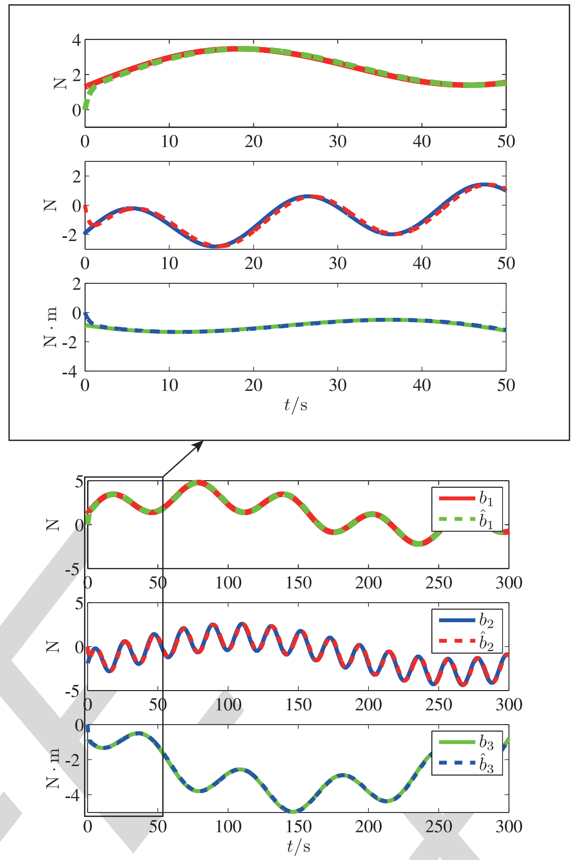

Fig. 8. Time-variant external disturbances b_1, b_2, b_3, and their estimations $\hat{b}_1$, $\hat{b}_2$, $\hat{b}_3$.

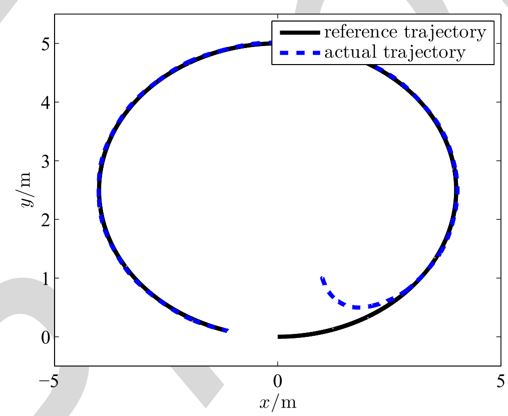

Fig. 9. Actual and reference trajectories in xy-plane under time-variant disturbances.

without disturbance observer

$$
\begin{aligned}
\tau_{\text{cm}} = &-(MS^T R^T C_{cm1} + R^T + C_{\text{cm2}} R^T C_{\text{cm1}})(\eta - \eta_d) \\
&+[M(S^T R^T + R^T C_{\text{cm1}}) + C_{\text{cm2}} R^T]\dot{\eta}_d + MR^T \ddot{\eta}_d \\
&+[C(\nu) + D(\nu) - MR^T C_{\text{cm1}} R - C_{\text{cm2}}]\nu \\
&-K_{\text{cm}} \int_0^t [\nu + R^T C_{\text{cm1}}(\eta - \eta_d) - R^T \dot{\eta}_d] d\eth \qquad (47)
\end{aligned}
$$

which is designed using the backstepping approach for the ship with constant disturbances in [10] with gains $C_{\text{cm1}} = \text{diag}[0.05, 0.05, 0.05]$, $C_{\text{cm2}} = \text{diag}[120, 120, 120]$,

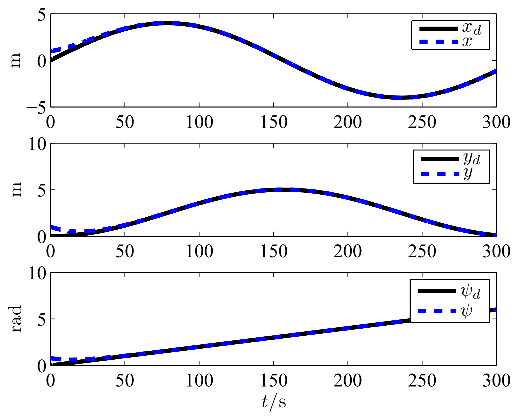

Fig. 10. Desired and actual positions and yaw angles under time-variant disturbances.

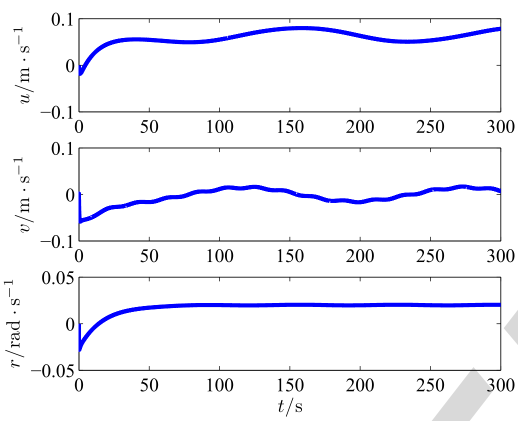

Fig. 11. Surge velocity u, sway velocity v, and yaw rate r under time-variant disturbances.

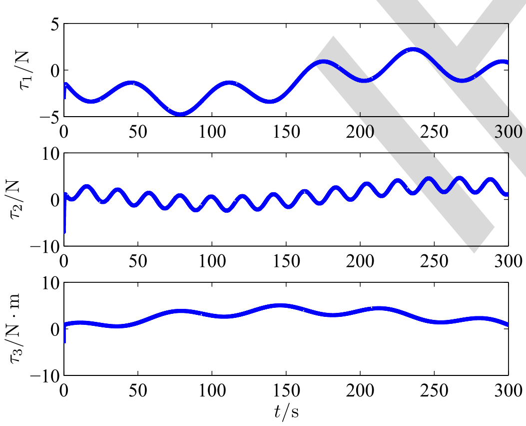

Fig. 12. Surge control force τ_1, sway control force τ_2, and yaw control torque τ_3 under time-variant disturbances.

and $K_{\text{cm}} = \text{diag}[2, 2, 2]$. Figs. 13 and 14 show the comparison of tracking performance between the two different controllers under constant disturbances and time-variant disturbances, respectively. It can be observed from Fig. 13 that both the controller exhibit similarly good transient and steady-state performances under the constant disturbances. Under time-variant disturbances, it is, however, observed from Fig. 14 that the controller τ with disturbance observer in

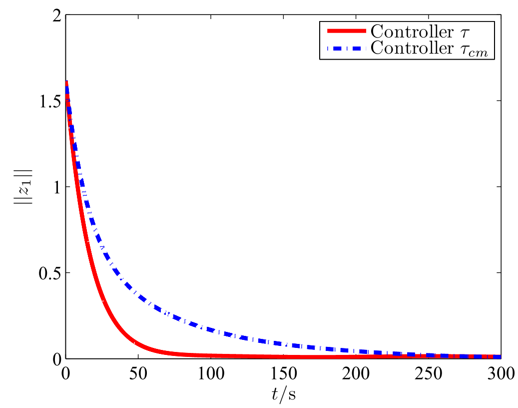

Fig. 13. Comparison of tracking performance under constant disturbances.

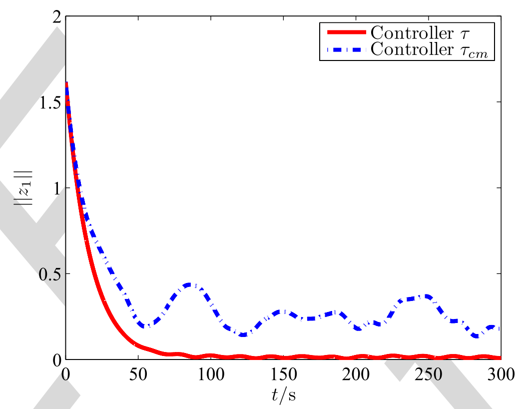

Fig. 14. Comparison of tracking performance under time-variant disturbances.

TABLE I
PERFORMANCE INDEX COMPARISON OF CONTROLLERS τ AND τ_{cm} UNDER DIFFERENT DISTURBANCES

Disturbances	Constant		Time-variant	
Controller	τ	τ_{cm}	τ	τ_{cm}
settling time t_s(s)	39	56	41	64
$\int_0^{t_{final}} \|x_e\| dt(\text{m} \cdot \text{s})$	19.4765	25.6207	19.8225	52.0868
$\int_0^{t_{final}} \|y_e\| dt(\text{m} \cdot \text{s})$	17.8455	40.7659	18.2010	55.9768
$\int_0^{t_{final}} \|\psi_e\| dt(\text{rad} \cdot \text{s})$	13.6354	35.1542	13.5816	44.8413

this brief performs better than the backstepping controller τ_{cm} with a faster decay of tracking error and lower steady-state error value because our observer provides an estimation of unknown disturbances. In contrast, τ_{cm} does not have disturbance compensation and results in a larger tracking error norm.

To quantitatively compare the two controller performance, the performance under both constant and time-variant disturbances is summarized in Table I, where $x_e = x_d - x$ and $y_e = y_d - y$ representing the error between the desired and actual positions, $\psi_e = \psi_d - \psi$ representing the error between the desired and actual yaw angles, and $t_{\text{final}} = 300$ s. Table I clearly shows that the controller τ has better steady state and transient performance than the backstepping controller τ_{cm}.

V. Conclusion

In this brief, a trajectory tracking robust control law has been designed for fully actuated surface vessels in the presence of uncertain time-variant disturbances due to wind, waves, and ocean currents. Both the Coriolis and centripetal matrix and the nonlinear damping terms have been considered in the nonlinear ship surface movement mathematical model. The control strategy is introduced by the vectorial backstepping technique with our disturbance observer. The disturbance observer is employed to compensate disturbance uncertainties. It has been proved that all the signals of the resulting closed-loop trajectory tracking system of the ship are globally uniformly ultimately bounded. Furthermore, the simulation results on an offshore supply ship model has illustrated that our controller is effective and robust to external disturbances. Our proposed trajectory tracking control scheme can provide good transient and steady-state performance for the considered ship system.

Future research would extend the proposed method to address the robust adaptive output feedback tracking of ships subjected to external disturbances and model uncertainties only depending on the position information $\eta = [x, y, \psi]^T$. From a practical viewpoint, it is convenient since it does not have to measure the velocities $\nu = [u, v, r]^T$ directly.

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